

# Prompt Nuclear EMP and Synchrotron Radiation: A Resolution of Two Approaches

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**Abstract**—The geomagnetic component of nuclear EMP generated by the prompt ( $t_{\max} \sim 2 \mu\text{s}$ )  $\gamma$ -ray, neutron, and X-ray radiation from a nuclear explosion has been the subject of intense scrutiny for over 40 years. Recent work by certain members of the scientific community has suggested that a discrepancy exists in the calculations/derivation of nuclear EMP between treatments based on Maxwell's equations and the high-frequency approximation and those derived from a summation over particles emitting synchrotron radiation. In principle, the two approaches should be identical simply because the well-known Liénard–Wiechert potentials for accelerating particles are derived from Maxwell's equations. In this paper, we start from the Liénard–Wiechert potentials and derive an expression for nuclear EMP that is identical to previous work based on a solution of Maxwell's equations. Thus, the putative discrepancy between the two approaches is resolved and Maxwell's equations in this regard are again vindicated.

**Index Terms**—EMP radiation effects, Maxwell equations, nuclear explosions, synchrotron radiation.

## I. INTRODUCTION

OUR BASIC understanding of the physics of the electromagnetic pulse (EMP) generated by a nuclear explosion underwent substantial evolution throughout the latter part of the 1950s and finally reached maturity in the early 1960s with the initial publications of Karzas and Latter [1]–[4] and the lecture notes of Longmire (cf. [5]–[7]). The essential features of EMP generation can be summarized as follows. The prompt radiation (X-rays,  $\gamma$ -rays, and neutrons) from a nuclear explosion ionizes the air and produces a temporally changing current system that launches an EMP. The current system consists of two parts: 1) energetic electrons ( $\sim 1\text{-MeV}$  energy) produced by Compton scattering of the  $\gamma$ -rays and moving radially outward and 2) secondary electrons that are produced by both X-rays (photoelectric effect) and ionization of the air by the Compton electrons. At low altitudes, X-rays are easily absorbed in air and are confined to a small region (a few meters in size at sea-level air density) around the burst point and contribute little to the currents that generate EMP. For high-altitude bursts ( $>50\text{ km}$ ), the X-rays become important both in producing a current system that contributes to the EMP and in creating background ionization over a large range that can absorb some of the EMP. In contrast, the  $\gamma$ -rays produce significant ionization over a spherical region several hundred meters to kilometers in radius at sea level. Neutrons undergo inelastic scattering and capture in air and produce additional  $\gamma$ -rays (at later times) that, in turn, generate Compton electrons. If asymmetries exist in the driving

currents, then a pulse is recorded by a remote sensor. Asymmetries in the Compton current (energetic electrons) can be caused by either: 1) differential absorption of  $\gamma$ -rays resulting from the presence of the ground or ground asymmetry EMP (GAEMP), the weapon design, or the atmospheric density gradient or 2) the turning of the Compton electrons in the earth's magnetic field or geomagnetic EMP (GEMP). It is the latter form of EMP on which we focus in this paper. We will also limit our analysis to early times ( $\sim 2 \mu\text{s}$ ), the  $\gamma$ -ray source term, and to low-altitude bursts ( $<25\text{ km}$ ), but the basic conclusion will apply at high altitudes as well.

The generation of a coherent electromagnetic pulse by the turning of Compton/relativistic electrons in a magnetic field generally falls under the rubric of synchrotron radiation. Thus, the radiation field can be computed by using the Liénard–Wiechert potentials and by summing the corresponding radiated, vector electric field over the particles. Because the latter potentials are in turn derivable from Maxwell's equations it is also possible to obtain the radiation field directly from these equations and an appropriate prescription for the Compton current. We show below that the two techniques are identical under the same assumptions.

### A. Existing Formulation of Nuclear EMP

The following derivation conforms to the basic treatments of nuclear GEMP presented in previous papers [1]–[7]. The fluid approach used here differs only slightly from previous work and is used only to simplify the problem and for clarity.

The relevant fluid equations for primary electrons (Compton electrons) and secondary electrons (produced by primaries) can be written

$$\frac{\partial n_p}{\partial t} = -\nabla \cdot n_p \mathbf{v}_p + F_\gamma / \lambda_c - \nu_E n_p \quad (1)$$

$$\frac{\partial \mathbf{S}}{\partial t} = -\nabla \cdot \mathbf{S} \mathbf{v}_p - e n_p \left( \mathbf{E} + \frac{\mathbf{v}_p \times \mathbf{B}}{c} \right) - \nu \mathbf{S} + F_\gamma \gamma_p m_e \mathbf{v}_b / \lambda_c \quad (2)$$

$$\frac{\partial n_s}{\partial t} = -\nabla \cdot n_s \mathbf{v}_s - \alpha n_s + \nu_i n_s + \left( \frac{\varepsilon_p}{34} \right) \nu_E n_p \quad (3)$$

where  $n_p$  is the primary electron density,  $\varepsilon_p$  is the primary electron energy in electronvolts (eV),  $\mathbf{S} = m_e \gamma_p n_p \mathbf{v}_p$  is the momentum density of primary electrons moving with a mean velocity  $\mathbf{v}_p$ ,  $\gamma_p$  is the Lorentz factor,  $F_\gamma$  is the flux at a radius  $r$  and time  $t$  of  $\gamma$ -rays produced by the nuclear explosion,  $\lambda_c$  is the Compton attenuation length for  $\gamma$ -rays,  $\nu_E$  is the normalized energy loss rate (in  $\text{s}^{-1}$ ) for primary electrons,  $\nu$  is the total momentum loss rate which can be written as the sum of energy loss and scattering rates for primary electrons,  $\mathbf{E}$  is the

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self-consistent electric field,  $\mathbf{B}$  is the sum of the geomagnetic field and self-consistent magnetic field,  $\mathbf{v}_b$  is the mean velocity with which the primary electrons are born,  $n_s$  is the density of secondary electrons,  $\mathbf{v}_s$  is the mean drift velocity of secondary electrons,  $\alpha$  is the sum of the two-body dissociative plus three-body attachment rates in air, and  $\nu_i$  is the ionization rate for secondary electrons. The energy loss rate used in (1) actually corresponds to a particle loss rate in that the population of energetic particles is limited by definition to a specified energy range. Particles that decelerate past the lower energy boundary are lost to the population of Compton electrons at the normalized rate defined by  $\nu_E$ . We note that the quantities  $\mathbf{v}_s$ ,  $\alpha$ , and  $\nu_i$  are functions of the electric field and can be obtained from swarm data assuming that secondary electron collisions are sufficiently rapid as to bring their distribution function into an equilibrium defined by the electric field. The latter condition is generally met at low altitudes. For high-altitude bursts, inertial effects come into play in the source region and the secondary electron momentum equation must be solved along with (1)–(3). The last term in (3) represents the production rate of secondary electrons as a result of ionization by the primary electrons. The factor  $((\varepsilon_p)/(34))$  accounts for the fact that a primary electron expends approximately 34 eV for every ion pair it produces.

The  $\gamma$ -ray flux produced by a nuclear weapon can be written

$$F_\gamma = \frac{\eta Y}{\varepsilon} \frac{e^{-r/\lambda_c}}{4\pi r^2} S_\gamma(t - r/c) \quad (4)$$

where  $\eta$  is the  $\gamma$ -ray efficiency,  $Y$  is the total yield of the weapon,  $\varepsilon$  is the mean energy of the  $\gamma$ -rays, and  $S_\gamma$  is the normalized rate (an integration over time yields one) of production of  $\gamma$ -rays in units of quanta per second. We note that the form of (4) does not apply near the origin of the burst where  $F_\gamma$  must ultimately go to zero. In this formulation, we limit ourselves to distances large compared to the weapon dimensions. The total current density associated with the motion of the primary and secondary electrons can be written

$$\mathbf{j} = \mathbf{j}_p + \sigma \mathbf{E}$$

where  $\mathbf{j}_p$  is the primary electron current density and  $\sigma = n_s e^2 / m_e \nu$ , where  $m_e$  is the rest mass of the electron,  $-e$  is its charge, and  $\nu$  is the secondary electron scattering rate is the conductivity produced by the secondary electrons. An equation for the primary electron current is obtained directly from (2) to yield

$$\begin{aligned} \frac{\partial \mathbf{j}_p}{\partial t} = & -\mathbf{j}_p \cdot \nabla \mathbf{v}_p - \frac{\mathbf{j}_p}{\gamma_p} \mathbf{v}_p \cdot \nabla \gamma_p - \frac{\mathbf{j}_p}{\gamma_p} \frac{\partial \gamma_p}{\partial t} - \mathbf{v}_p \cdot \nabla \mathbf{j}_p \\ & + \frac{e^2 n_p}{\gamma_p m_e} \left( \mathbf{E} + \frac{\mathbf{v}_p \times \mathbf{B}}{c} \right) - \nu \mathbf{j}_p - F_\gamma e \mathbf{v}_b / \lambda_c \end{aligned} \quad (5)$$

where  $\mathbf{j}_p = -n_p e \mathbf{v}_p$ . An equation for the mean velocity of the primary electrons is obtained by combining (1) and (2)

$$\begin{aligned} \frac{\partial \mathbf{v}_p}{\partial t} = & -\frac{\mathbf{v}_p}{\gamma_p} \mathbf{v}_p \cdot \nabla \gamma_p - \frac{\mathbf{v}_p}{\gamma_p} \frac{\partial \gamma_p}{\partial t} - \mathbf{v}_p \cdot \nabla \mathbf{v}_p \\ & - \frac{e}{\gamma_p m_e} \left( \mathbf{E} + \frac{\mathbf{v}_p \times \mathbf{B}}{c} \right) - \nu_s \mathbf{v}_p + F_\gamma (\mathbf{v}_b - \mathbf{v}_p) / n_p \lambda_c \end{aligned} \quad (6)$$

where  $\nu_s$  is the primary electron scattering rate.

In order to simplify the problem, we now transform (5) into the frame moving at the speed of light in the radial direction with the  $\gamma$ -rays. We introduce the variables  $\tau = (t - r/c)$  and  $r' = r$ . We also employ the high-frequency approximation which assumes that the time for production of the  $\gamma$ -rays multiplied by the speed of light defines a scale length (Compton shell size) that is much shorter than the Compton attenuation length  $\lambda_c$  for low-altitude bursts and shorter than the atmospheric scale height for high-altitude bursts (c.f. [4] and [8]). Typically, the Compton shell size is  $< 3$  m, while the Compton attenuation length at sea level is  $\lambda_c \sim 300$  m. Under this approximation

$$\left| \frac{(1 - \beta_{pr})}{c} \frac{\partial}{\partial \tau} \right| \gg |\nabla'| \quad (7)$$

where  $\beta_{pr}$  is the primary electron radial velocity divided by the speed of light. We note that in the moving frame the production time of primary electrons is shortened by the factor  $(1 - \beta_{pr})$  so that the appearance of this factor in front of the  $\tau$  derivative does not change the requirement for the validity of (7) as stated above.

The transformation of (5) yields

$$\begin{aligned} (1 - \beta_{pr}) \frac{1}{\gamma_p} \frac{\partial \gamma_p \mathbf{j}_p}{\partial \tau} - \frac{\partial \beta_{pr}}{\partial \tau} \mathbf{j}_p = & -\nu \mathbf{j}_p - F_\gamma e \mathbf{v}_b / \lambda_c \\ & + \frac{e^2 n_p}{\gamma_p m_e} \left( \mathbf{E} + \frac{\mathbf{v}_p \times \mathbf{B}}{c} \right) \end{aligned} \quad (8)$$

To obtain a solution of (8), we first make the assumptions that  $\mathbf{v}_b = v_0 \hat{\mathbf{r}}$ ,  $|\mathbf{v}_p| = v_0$ , and  $v_0$  (and, therefore,  $\gamma_p$ ) is independent of  $\tau$ . These assumptions are valid provided that the primary electrons are produced with a mean velocity mainly in the radial direction (consistent with Compton scattering) and that the production rate of particles is much faster than any of the transport processes (consistent with the high-frequency approximation). We must also assume that the acceleration of particles due to the electric field is small. The maximum field is the radial component that saturates due to the canceling effect of the secondary electron conductivity. According to Longmire [9] the maximum field (independent of yield but proportional to the production rate of  $\gamma$ -rays plus the three-body attachment rate of secondary electrons) is 60 kV/m at sea level and 40 kV/m at high altitudes for a production rate of  $2 \times 10^8 \text{ s}^{-1}$ . These fields are sufficiently small to neglect their effect on the speed  $v_0$  of the primary electrons relative to the rate at which particles are born with speed  $v_0$ . The net result of these approximations is that the average energy of the Compton electrons remains constant for any  $r$  and  $\tau$  and equal to the energy at which the particles are born. The particle population however is still reduced by energy loss due to collisions albeit at a rate that is small compared to the production rate. Thus, (8) now reduces to

$$(1 - \beta_{pr}) \frac{\partial \mathbf{j}_p}{\partial \tau} = -\nu \mathbf{j}_p - F_\gamma e v_0 \hat{\mathbf{r}} / \lambda_c - \Omega \mathbf{j}_p \times \hat{\mathbf{B}} \quad (9)$$

where  $\hat{\mathbf{B}}$  is the vector direction of the magnetic field and  $\Omega = (eB)/(\gamma_p m_e c)$ . For simplicity, we adopt the description of collisions used by Karzas and Latter [4], namely that the primary electrons are effectively collisionless throughout their

range and then are abruptly lost. This formulation allows us to drop the  $(\nu j_p)$  term in obtaining a solution and then reincorporate it as an integral limit in the final solution. Solving (8) under these approximations for the various vector components of the primary current and taking the magnetic field to be in the  $z$ -direction (without loss of generality) yields

$$j_{pr}(r, \tau) = \frac{-ev_0}{\lambda_c} \int_0^{\tau'} \frac{d\tau''}{(1 - \beta_{pr})} F_\gamma(r, \tau - \tau'') \times (\cos^2 \theta + \sin^2 \theta \cos \Omega' \tau'') \quad (10)$$

$$j_{p\theta}(r, \tau) = \frac{-ev_0}{\lambda_c} \int_0^{\tau'} \frac{d\tau''}{(1 - \beta_{pr})} F_\gamma(r, \tau - \tau'') \cdot \cos \theta \sin \theta (\cos \Omega' \tau'' - 1) \quad (11)$$

$$j_{p\phi}(r, \tau) = \frac{-ev_0}{\lambda_c} \int_0^{\tau'} \frac{d\tau''}{(1 - \beta_{pr})} F_\gamma(r, \tau - \tau'') \cdot \sin \theta \sin \Omega' \tau'' \quad (12)$$

where  $\Omega' = \Omega/(1 - \beta_{pr})$ . We have dropped the prime on the  $r$ -variable,  $\tau' = (1 - \beta_{pr})/\nu$ , and  $\theta$  is the polar angle between the magnetic field and the radial direction. These results are identical to (14)–(16) of Karzas and Latter [4] in the limit that  $\Omega' \tau' \ll 1$  which is accurate at low altitudes. At high altitudes, the results are also identical along the rise of the  $\gamma$ -pulse. At later times, the two results differ in the argument of  $F_\gamma$ .

An expression for the secondary electron density can be obtained from (1) and (3). After transforming to the  $\gamma$ -frame, we find

$$n_s(r, \tau) = \frac{\nu E}{\lambda_c} \left( \frac{\varepsilon_p}{34} \right) \int_0^\tau d\tau' e^{(\nu_i - \alpha)(\tau - \tau')} \cdot \int_0^{\tau'} \frac{d\tau''}{(1 - \beta_{pr})} F_\gamma(\tau'', r) e^{-\nu E (\tau' - \tau'')/(1 - \beta_{pr})}. \quad (13)$$

This result agrees with (13) of Karzas and Latter [4] in the same limits discussed previously for the primary current. Note that we have incorporated the effects of secondary ionization and attachment in our solution. Price [8] modified the results of Karzas and Latter [4] to include attachment.

In the high-frequency approximation, the appropriate equations for the electric field components are derived in both Karzas and Latter [4] and, more recently, in Price [8]. The relevant equations are written

$$\frac{\partial E_r}{\partial \tau} = -4\pi(j_{pr} + \sigma E_r) \quad (14)$$

$$\frac{1}{r} \frac{\partial r E_\theta}{\partial \tau} = -\frac{2\pi}{c} (j_{p\theta} + \sigma E_\theta) \quad (15)$$

$$\frac{1}{r} \frac{\partial r E_\phi}{\partial \tau} = -\frac{2\pi}{c} (j_{p\phi} + \sigma E_\phi). \quad (16)$$

The radial component of the electric field  $E_r$  represents a quasi-electrostatic field formed by the separation in charge between the primary electrons and the ions produced by primary ionization and cancelled in part by the secondary current. The

solution for the radiated fields,  $E_\theta$  and  $E_\phi$ , can be written

$$E_{\theta,\phi}(r, \tau) = -\frac{2\pi}{rc} \int_0^\tau dr' r' j_{p\theta,\phi}(r', \tau) e^{-\frac{2\pi}{c} \int_{r'}^\tau dr'' \sigma(r'', \tau)}. \quad (17)$$

Equations (10)–(13) together with (17) form a complete set and provide a general expression for the electric field radiated by a nuclear explosion in the high-frequency approximation. We will derive the same expression below starting from the Liénard–Wiechert potentials.

## II. STARTING FROM THE LIÉNARD–WIECHERT POTENTIALS

The Liénard–Wiechert potentials for a particle in motion are derivable from Maxwell's equations and can be written [10]

$$\Phi(\mathbf{x}, t) = \left[ \frac{e}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})R} \right]_{\text{ret}} \quad \mathbf{A}(\mathbf{x}, t) = \left[ \frac{e\boldsymbol{\beta}}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})R} \right]_{\text{ret}} \quad (18)$$

where the subscript “ret” means that the quantity in the square brackets is to be evaluated at the retarded time,  $\boldsymbol{\beta}$  is the particle velocity divided by the speed of light,  $\mathbf{n}$  is the unit vector from the particle to the observer, and  $R$  is the distance from the particle to the observer. From these potentials, it is possible to derive the following expressions for the corresponding electric and magnetic fields:

$$\mathbf{E}(\mathbf{x}, t) = e \left[ \frac{(\mathbf{n} - \boldsymbol{\beta})}{\gamma^2 (1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R^2} \right]_{\text{ret}} + \frac{e}{c} \left[ \frac{\mathbf{n} \times \left\{ (\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} \right\}}{(1 - \boldsymbol{\beta} \cdot \mathbf{n})^3 R} \right]_{\text{ret}} \quad (19)$$

$$\mathbf{B}_{\text{rad}} = [\mathbf{n} \times \mathbf{E}]_{\text{ret}}.$$

The first term in the expression for  $\mathbf{E}(\mathbf{x}, t)$  corresponds to a quasi-electrostatic field, while the second term represents the radiation produced by an accelerating charge with  $\dot{\boldsymbol{\beta}}$  equal to the particle acceleration. Synchrotron radiation for a single particle is derived from this term and is the focus of our analysis.  $\mathbf{B}_{\text{rad}}$  represents the radiated magnetic field.

We note that the Liénard–Wiechert potentials only address the radiation produced by accelerating charges. The creation of charge in a way that results in a changing current also yields radiation and this effect must be included separately provided that a change in this current is not produced by the magnetic field in which case the Liénard–Wiechert potentials do include this particular component. In the context of nuclear EMP, the  $\gamma$ -rays produce a changing Compton current that in principle yields net radiation provided that an asymmetry in the current system exists. We will show below that the geomagnetic field introduces such an asymmetry and that there is net radiation produced as a result. This radiation is in addition to that produced by the acceleration of the individual electrons.

In order to proceed, we first adopt the collision model used by Karzas and Latter [4], i.e., a particle is collisionless up to the

collision time and then stops abruptly. With this approximation, we have

$$\dot{\beta}^{\dot{\gamma}} = \frac{e\beta \times \mathbf{B}}{\gamma mc} \quad (20)$$

where  $\mathbf{B}$  is the self-consistent magnetic field plus the externally applied field (e.g., geomagnetic field). The contribution to the radiation field from frictional deceleration (Bremsstrahlung radiation) is incoherent and is therefore small compared to the synchrotron term after summing over the particles. To remain consistent with the assumptions (and our collision model) made in the previous section, we neglect this effect and we also omit the acceleration of the Compton electrons by the self-consistent electric field. The radiation produced by acceleration of the secondary electrons by the electric field is also omitted from the Liénard–Wiechert potentials. Instead the secondary electron radiation is treated below as resulting from a macroscopic conduction current (proportional to the electric field) that in effect “absorbs” the synchrotron radiation produced by the Compton electrons. In a more rigorous particle treatment, we would have to include this effect by means of a secondary electron acceleration term proportional to the electric field and incorporated in the Liénard–Wiechert potentials. Our contention is that the radiation derived in this way and summed over all secondary electrons would serve to cancel in part that produced directly by synchrotron radiation and would appear therefore exactly as an absorption term in the final expression for the net radiation. The proof will be left for future work but the analysis performed here for synchrotron radiation provides the methodology for treating the secondary electrons and lends support to our contention in this regard.

If we make the further assumption that particles are moving primarily in the radial direction (justified *a posteriori*) then the expression for the radiated component of the electric field becomes

$$\mathbf{E}(\mathbf{x}, t) = \frac{e}{c} \left[ \frac{\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \dot{\beta}^{\dot{\gamma}})}{(1 - \beta)^2 R} \right]_{\text{ret}} \quad (21)$$

Note that the retarded frame now corresponds with the  $\gamma$ -frame. Substituting the solution to (20) into (21) and assuming that the magnetic field is in the  $z$ -direction (again, without loss of generality) we have

$$\mathbf{E}(\mathbf{x}, \tau - \tau') = \frac{e}{c} \left[ \begin{aligned} & \frac{\beta \Omega' \sin \theta}{(1 - \beta) R} \\ & \cdot \left\{ \begin{aligned} & \cos \Omega'(\tau - \tau') \hat{\phi} \\ & + \cos \theta \sin \Omega'(\tau - \tau') \hat{\theta} \end{aligned} \right\} \\ & \cdot \left\{ H \left( \tau' - \tau + \frac{(1 - \beta)}{\nu_E} \right) - H(\tau' - \tau) \right\} \end{aligned} \right]_{\text{ret}} \quad (22)$$

where  $\Omega' = eB/\gamma mc (1 - \beta)$  and  $\tau - \tau'$  is a time interval in the  $\gamma$ -frame described in the previous section. To be consistent with our adopted collision model, this time interval cannot exceed a value equal to  $(1 - \beta)/\nu_E$ , where  $\nu_E$  is the energy loss rate. We have imposed this condition mathematically by means of the Heaviside functions  $H(\tau)$ .

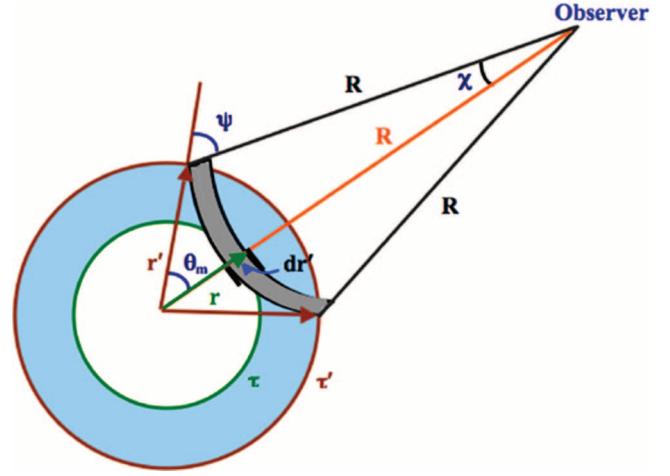


Fig. 1. Geometry for EMP production. The light blue color represents the  $\gamma$ -shell between  $\tau$  and  $\tau'$  in the  $\gamma$ -frame or  $r$  and  $r'$  in the fixed frame. The Compton electrons with density  $n_p(r', \tau - \tau')$  on the spherical cap (in three dimensions) centered on the observer and with radius  $R$  radiate an EMP that reaches the observer at time  $\tau$ . The spherical cap is swept forward by the motion of the  $\gamma$ -shell over a distance  $dr'$  as shown in the picture. As the shell moves out, radiation piles up in the grey volume element provided absorption by the conducting gas is sufficiently small. This condition is generally satisfied at radial distances greater than a critical value  $r_c$  equal to several  $\gamma$ -attenuation lengths or approximately 1 km for a low-altitude burst. The contribution of the Compton electrons must be summed or integrated from  $r_c$  to the observer and over the lifetime of the Compton electrons to obtain the total EMP.

The electric field  $\mathbf{E}$  given by (22) represents the radiated field from a single particle moving primarily in the radial direction and accelerated (without change of energy) by the total magnetic field  $\mathbf{B}$ . To obtain the net radiation from an ensemble of particles, we must sum over the total number of particles that emit coherently within an appropriate volume element. In general, the total number of particles  $dN$  in a volume  $dV'$  can be written very simply as

$$dN = n_p(r', \tau') dV' \quad (23)$$

where  $n_p$  is the density of primary electrons at  $\tau'$  and  $r'$ . The problem now reduces to defining the correct volume over which to count the particles that contribute to the radiation that arrives at the observer at time  $\tau$  (measured relative to the start of the  $\gamma$ -radiation from the burst).

The appropriate geometry is illustrated in Fig. 1. This figure shows two spherical surfaces of constant  $\tau$  that, at a given time  $t$  in the fixed frame, coincide with the radial surfaces,  $r$  and  $r'$ , with  $\Delta r = r' - r = c(\tau - \tau')$ . In general, the  $\tau$  surfaces move outward at the speed of light and cross surfaces of increasing radius as a function of the fixed-frame time coordinate  $t$ . Note also that  $\tau$  coincides with a particular time into the  $\gamma$ -ray pulse [function  $S_\gamma$  in (4)] produced by the nuclear burst and that its value decreases with radial distance at any given time  $t$ . Thus, for example,  $\tau'$  is less than  $\tau$  in Fig. 1.

In order to arrive simultaneously at the observer, the radiation produced in the source region must originate along the spherical surface centered at the observer and of radius  $R$ . The Compton electrons that lie on the surface (in the source region) outline a spherical cap that is tangent to the surface  $\tau$  and that crosses the surface  $\tau'$ . The spherical cap moves forward in time along with

the  $\gamma$ -pulse from the burst and radiates. The angle  $\chi$  shown in Fig. 1 is measured from the line-of-sight line segment to the line that connects the location of the observer to the point where the  $\tau'$  surface crosses the spherical cap. The surface area  $dA$  of the spherical cap can be written in terms of  $\chi$  and  $R$  as

$$dA = \pi R^2 \chi^2 \quad (24)$$

where  $R$  is the distance to the observer as before and where we have assumed  $\chi \ll 1$ . Given the radial distance  $\Delta r = c(\tau - \tau') = cd\tau'$  and assuming that  $(\Delta r/r') \ll 1$  (justified below), the angle  $\chi$  can be approximated as  $\chi \approx \sqrt{(2r'\Delta r/R^2)}$  and the volume element  $dV'$  becomes

$$dV' = 2\pi r' \Delta r dr' = 2\pi r' cd\tau' dr' \quad (25)$$

where we have taken into account the fact that the spherical cap is swept forward a radial distance  $dr'$  by the motion of the  $\gamma$ -shell as noted previously.

The maximum value of  $\Delta r$ ,  $\Delta r_{\max} = c(1-\beta)/(\nu_E)$ , is defined by our model for particle collisions and the fact that our result (22) for the radiated field from one particle is limited to a time interval equal to the lifetime of the Compton electron. Only those particles within the corresponding range of  $\tau$  values,  $\tau - \tau' \leq (1-\beta)/(\nu_E)$  can emit radiation coherently and with the relative phase defined in (22). Those particles that lie outside of this  $\tau$  interval possess retarded times that are larger than the lifetime of the particle and therefore their trajectories are such that they die before they can contribute to the radiation. Another way to view this result is to realize that the lifetime of the particles defines a correlation time (in the  $\gamma$ -pulse frame) over which the memory of particles produced during their lifetime results in a spread of information or mathematical convolution of the source term across that interval of time. The resulting convolution will be evident in the final results in the following. We note that this effect also limits the angular range of  $\chi$  and therefore the extent of the spherical cap.

The value of  $\Delta r_{\max}$  at sea level is approximately equal to 0.24 m for a 1-MeV primary electron. The source region (defined to start at the radial distance where the radiation can escape to the observer) for EMP lies beyond the radial distance of  $\sim 1$  km from the burst point for low-altitude bursts. Thus, the inequality,  $(\Delta r)/(r') \ll 1$ , is readily satisfied. Using these values, we also find that the maximum value of the polar angle as illustrated in Fig. 1 is  $\theta_m \approx 1.25^\circ$ , a value that is much less than the cone angle  $\theta_s$  for synchrotron emission for a 1-MeV primary electron ( $\theta_s \sim 20^\circ$ ). Because the angle  $\psi$  (see Fig. 1) is approximately equal to  $\theta_m$ , this result demonstrates that we are justified in assuming that the radiation is forward directed and that the radiating particles are moving primarily in the radial direction. We note that these same conditions apply for high-altitude bursts.

Combining (22)–(25), integrating over  $\tau'$  from 0 to  $(1-\beta)/(\nu_E)$  and  $r'$  from 0 to  $R$ , and incorporating the effect of absorption due to the background conductivity created by the secondary electrons (as discussed previously), the total radiated field due to primary electrons ( $e \rightarrow -e$ , and  $\Omega' \rightarrow -\Omega'$ ) can be

written

$$E_\phi(r, \tau) = -\frac{2\pi}{Rc} ev_0 \Omega' \int_0^R dr' r' e^{-\frac{2\pi}{c} \int_{r'}^r dr'' \sigma(r'', \tau)} \cdot \int_0^{\tau_s} d\tau' n_p(r', \tau - \tau') \cos \Omega' \tau' \sin \theta \quad (26)$$

$$E_\theta(r, \tau) = \frac{2\pi}{Rc} ev_0 \Omega' \int_0^R dr' r' e^{-\frac{2\pi}{c} \int_{r'}^r dr'' \sigma(r'', \tau)} \cdot \int_0^{\tau_s} d\tau' n_p(r', \tau - \tau') \sin \Omega' \tau' \sin \theta \cos \theta \quad (27)$$

where  $\tau_s = (1-\beta)/(\nu_E)$ . Equations (26) and (27) can be simplified by writing

$$\Omega' \cos \Omega' \tau' = \frac{d}{d\tau'} \sin \Omega' \tau', \quad \Omega' \sin \Omega' \tau' = -\frac{d}{d\tau'} \cos \Omega' \tau',$$

integrating by parts and substituting

$$\frac{d}{d\tau'} n_p(r', \tau - \tau') = -\frac{F_\gamma}{(1-\beta)\lambda_c}.$$

The result is

$$E_\phi(r, \tau) = \frac{2\pi}{Rc} \frac{ev_0}{\lambda_c} \int_0^R dr' r' e^{-\frac{2\pi}{c} \int_{r'}^r dr'' \sigma(r'', \tau)} \cdot \int_0^{\tau_s} \frac{d\tau'}{(1-\beta)} F_\gamma(r', \tau - \tau') \sin \Omega' \tau' \sin \theta \quad (28)$$

$$E_\theta(r, \tau) = \frac{2\pi}{Rc} \frac{ev_0}{\lambda_c} \int_0^R dr' r' e^{-\frac{2\pi}{c} \int_{r'}^r dr'' \sigma(r'', \tau)} \cdot \int_0^{\tau_s} \frac{d\tau'}{(1-\beta)} F_\gamma(r', \tau - \tau') \cos \Omega' \tau' \sin \theta \cos \theta. \quad (29)$$

Equation (28) is identical to (12) and (17) of the previous section for the  $\phi$ -component of the radiated electric field. This component is associated strictly with the acceleration of the Compton electrons by the geomagnetic field. The  $\theta$ -component arises from both the acceleration and the asymmetry introduced by the geomagnetic field in the changing Compton current. An additional term needs to be added to (29) to accommodate this effect.

As noted previously the Liénard–Wichert potentials account for radiation produced by accelerating charges. In the absence of an accelerating force such as an external magnetic field, the temporally changing Compton current would lie along the radial direction and would produce zero net radiation for a spherical explosion. In this case, the  $z$ -component of the Compton current separately produces a radiated field in the  $\theta$ -direction that is exactly canceled by the contribution from the radial current in the plane perpendicular to the magnetic field, i.e., the  $\rho$ -direction (see Fig. 2). In the particle approach, it turns out that the  $\rho$ -component is equivalent [after integration by parts; see (26)–(29)] to radiation resulting from the change in current associated with the production of Compton electrons coupled to a rotation of the current (introduces the phase term  $\cos \Omega' t'$ ) in the presence of the magnetic field. This component

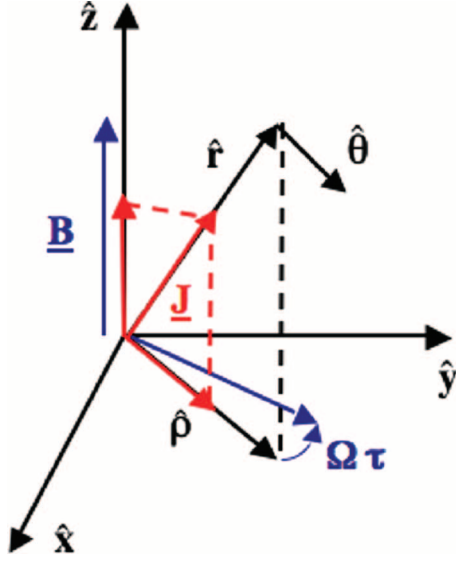


Fig. 2. Asymmetry in Compton current introduced by geomagnetic field. In the absence of an accelerating force such as an external magnetic field, the temporally changing Compton current would lie along the radial direction (as shown in red) and would produce zero net radiation for a spherical explosion. In this case, the  $z$ -component of the Compton current produces a radiated field in the  $\theta$ -direction that is exactly canceled by the contribution from the radial current in the  $\rho$ -direction. In the particle approach, it turns out that the  $\rho$ -component is equivalent [after integration by parts; see (26)–(29)] to radiation resulting from the change in current associated with the production of Compton electrons coupled to a rotation of the current (introduces the phase term  $\cos \Omega t'$ ) in the presence of the magnetic field. This component leads to a corresponding contribution to the radiation in the  $\theta$ -direction given by (29). To obtain the total (net) radiation in the  $\theta$ -direction, we must also include the changing current in the  $z$ -direction due to the production of Compton electrons just as we would in the case of no magnetic field. This term is not accounted for by the particle approach because there is no acceleration in the direction of the magnetic field. Ultimately, one can think of the  $\theta$ -component of the radiated electric field as resulting from the asymmetry introduced by the magnetic field in the production of Compton electrons.

leads to a corresponding contribution to the radiation in the  $\theta$ -direction given by (29). To obtain the total (net) radiation in the  $\theta$ -direction, we must also include the changing current in the  $z$ -direction due to the production of Compton electrons just as we would in the case of no magnetic field. This term is not accounted for by the particle approach because there is no acceleration in the direction of the magnetic field. Ultimately, one can think of the  $\theta$ -component of the radiated electric field as resulting from the asymmetry introduced by the magnetic field in the production of Compton electrons while the true synchrotron acceleration component (as we are accustomed to viewing it for an individual electron) is reflected in the  $\phi$ -component. We should not lose sight of the fact however that it is the particle acceleration caused by the magnetic field that produces the  $\rho$ -component of the radiation and that this component happens to be equivalent to radiation resulting from the change in current associated with the production of Compton electrons.

The  $z$ -contribution to the radiation field can be derived from (9) and (11) and (17) of the previous section. The final result for the total radiated field in the  $\theta$ -direction is

$$E_\theta(r, \tau) = \frac{2\pi}{Rc} \frac{ev_0}{\lambda_c} \int_0^R dr' r' e^{-\frac{2\pi}{c} \int_{r'}^r dr'' \sigma(r'', \tau)}$$

$$\cdot \int_0^{\tau_s} \frac{d\tau'}{(1-\beta)} F_\gamma(r', \tau - \tau') (\cos \Omega' \tau' - 1) \sin \theta \cos \theta \quad (30)$$

The results, (28) and (30), are identical to (11), (12), and (17). We note that when  $\Omega' = 0$  the radiated fields are zero as required, i.e., no net radiation is produced by a spherically symmetric current system.

### III. DISCUSSION

We have shown that under the same limits and approximations, the EMP produced by a nuclear explosion can be derived either from a solution of Maxwell's equations coupled to a fluid treatment of primary and secondary electrons or from the Liénard–Wiechert potentials coupled to a particle representation for the primary electrons. In order to achieve agreement between the two approaches, it is essential to count the number of emitting particles correctly and to include only those that emit coherently within a time span equal to the lifetime of the particles and only those whose radiation arrives at the detector at the same time. The latter constraints define an emitting volume (equivalent to a total number of radiating particles) that is consistent with the high-frequency approximation discussed in Section II. In addition, the sum of the synchrotron-radiated fields from these particles yields the same radiation at a remote detector as that computed directly from Maxwell's equations and the corresponding macroscopic particle currents.

The particle treatment we utilized in this paper only incorporates synchrotron radiation and does not include radiation resulting from frictional losses (Bremsstrahlung radiation) or acceleration by the electric field. This analysis is consistent with the assumptions that were made in the treatment based on the fluid/Maxwell equations. In a rigorous particle treatment of the entire nuclear EMP problem the secondary electron acceleration due to the electric field would have to be incorporated into the Liénard–Wiechert potentials. Instead, the secondary electron radiation is treated in Section III as resulting from a macroscopic conduction current (proportional to the electric field) that in effect “absorbs” the synchrotron radiation produced by the Compton electrons. The more rigorous approach will be left for future work and should be a straightforward extension of the methodology described in this manuscript.

We emphasize that we have only addressed the comparison between fluid/Maxwell equations versus particle/synchrotron radiation approaches and that the validity of the results rests entirely on the accuracy of the high-frequency approximation. The latter issue has been analyzed in more detail in previous reports [4], [8], [11]. The technique adopted by Longmire [7] of separating the fields into ingoing and outgoing components also provides a physically transparent confirmation of the high-frequency approximation. Studies of the EMP produced in a two-dimensional geometry or with corrections for transverse gradients along multiple rays from the source to the observer have been performed [7], [12]–[14] and resulted in further confirmation of the dominance at early times (<several microseconds) of the results obtained with the high-frequency approximation.

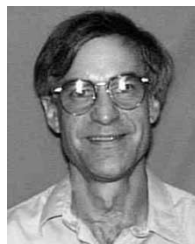
Although further work is needed, particularly in comparing with available data, our ability to predict the EMP from a nuclear explosion is well established and the basic physical principles are well known and modeled. Any new numerical treatments (including those based on Monte Carlo calculations of synchrotron radiation) should be judged on the basis of their ability to reproduce existing model results. Finally, from a purely heuristic perspective, it is important to note that the equivalence obtained in this paper between the two approaches (sum over individual particle emissions versus a coherent representation by means of macroscopic currents) has relevance to all calculations of coherent electromagnetic radiation from a particle source.

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